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Stability and Resilience of Transportation Systems: Is a Traffic Jam About to Occur?

Amin Ghadami, Charles R. Doering, John M. Drake, Pejman Rohani, and Bogdan I. Epureanu

***Abstract*— Measurement of traffic flow stability and resilience is a critical step toward evaluating the performance of transportation systems and implementing appropriate management strategies. Quantifying changes in the stability and resilience of transportation systems, however, is hampered by the complexity of real traffic dynamics and the diversity of infrastructures. Here, we demonstrate that changes in traffic flow stability and resilience are signaled by generic features, known as early warning signals in the theory of critical slowing down, observed before traffic instabilities occur. This finding is incorporated in an operational data-driven algorithm to evaluate the risk of traffic jams on highways. Theoretical findings and tests on simulated and empirical case studies support the premise of this approach and identify candidate statistical measures that are sensitive to changes in the stability and resilience of transportation systems. Our use of universal measures advances the monitoring capability, prediction and control of complex transportation systems.**

***Index Terms*— Early warning signals, resilience, traffic congestion, tipping point, complex system.**

I. Introduction

CONTINUOUS growth of the number of motor vehicles has made traffic congestion a serious problem around the world. Traffic congestion is intertwined with other concerns that significantly affect quality of life, ranging from economic and environmental problems to behavioral and health consequences related with vehicle emission [1]–[3]. Recent concepts related to smart cities and flexible, multi-modal transportation further increase the complexity of the dynamics of transportation systems of the future. Hence, the study of the complex dynamics of traffic has emerged as a topic of considerable attention from engineers, planners, and policymakers [4]–[6], with the ultimate goal of improving the stability and resilience of transportation systems by real-time traffic control management. However, to implement smart control of road traffic, predicting traffic conditions and developing universal measures of traffic stability and resilience are a prerequisite.

Predictions and evaluations based on empirical models is one of the traditional approaches in traffic technology to analyze traffic patterns [7]–[9]. These models have provided invaluable understanding of vehicular traffic systems and the underlying dynamics resulting in traffic congestions. However, practical real-time application of these approaches face significant challenges, particularly the necessity to approximate and to validate model parameters corresponding to real-time traffic conditions [10], [11]. Recently, availability of traffic data have motivated data-assisted and data-driven techniques to improve the existing methods and address existing challenges in traffic jam analysis and forecasting [12]–[14]. Despite great advantages of these methods, they usually demand extensive data for model building and prediction. In addition, the applicability of the methods remains a question in places where sufficient data availability is an issue [15]. As a result, what is needed is a methodology that connects data-driven and parametric approaches. Here, we present such a framework by introducing universal measures of traffic performance to capture, monitor, and evaluate changes in the system stability and resilience over time regardless of availability of any detailed information about the underlying system model and parameters.

Previous studies have confirmed that traffic jams are the result of an instability and phase transition (i.e., rapid transition to a new equilibrium state) in the traffic flow dynamics where the traffic dynamics switches from a homogeneous flow state to a congested flow state [16]–[18]. For instance, if the density of vehicles on a road exceeds some critical value or a change in driving strategies occurs depending on time and weather conditions, the initially homogeneous traffic loses its stability resulting in traffic jams that move upstream [16], [19]–[21]. Recent advances in complex systems theory offer a hope to identify footprints of critical phenomena in dynamical systems, e.g., instabilities and rapid change in equilibrium state, solely from recorded measurements of the system dynamics during normal operational conditions [22]–[25]. When a dynamical system approaches a tipping point – a threshold that when exceeded leads to instability or large changes in the state of the system – its dynamics become progressively slow, a phenomenon known as critical slowing down. As a consequence of the slowing down phenomenon, perturbations to the system lead to longer transient recoveries to the equilibrium state, and the system

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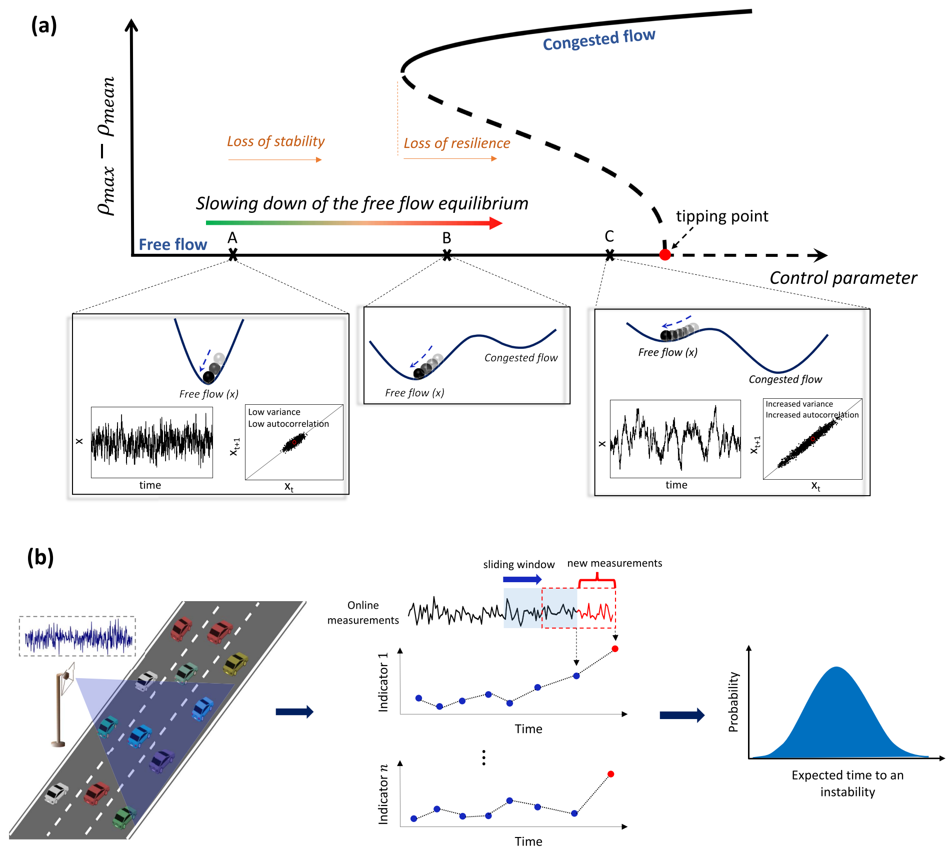

(i) On-line measurements of the traffic dynamics

(ii) Calculation of the early warning indicators

(iii) Evaluation of the traffic stability

Fig. 1. (a) Schematic of the critical slowing down phenomenon illustrated for a traffic flow system approaching a tipping point. Panels A, B and C show the behavior of the system around its uniform flow equilibrium state while approaching an instability (i.e., a traffic jam). The local minima of the potential well represent stable attractors, and the ball shows the present state of the system. While approaching the critical parameter, the local minimum of the stable equilibrium becomes shallower, and the recovery of the ball in response to small perturbations is progressively slowing down. The footprint of this slowing down is observed in the statistical measures obtained from measured time series of the dynamics. When the local minimum finally disappears, the system undergoes a critical transition from its initial equilibrium (free flow) to a new equilibrium state (congested flow). In the diagram, $\rho_{max} - \rho_{mean}$ reflects the equilibrium state, where ρ_{mean} is the average car density on road and ρ_{max} is the maximum observed local car density over of the road in a steady state situation. (b) Schematic of the proposed method of early warning signals for online applications in vehicular traffic systems. Statistical indicators of traffic flow instability are extracted from measured traffic dynamics (e.g., average driving speeds) on the road, and they are analyzed to assess the traffic stability and resilience, the effectiveness of adopted traffic management strategies, and the risk of an upcoming congestion.

is critically slow at the tipping point [26] (Fig. 1).Hence, probing for the signs of slowing down accompanying critical transitions can be employed as a basis for novel data-driven approaches to anticipate stability, resilience and emergence of critical phenomena in transportation systems. A detectable slowing down can be interpreted as an increased risk of instability and loss of resilience in the sense that the traffic flow system could easily be tipped into a congested state.

In order to test whether a system undergoes slowing down, one would ideally investigate its response to perturbations from time to time, although such an approach may not be feasible in practice [23], [27]. As an alternative, the footprints of slowing down in dynamical systems can be investigated via trends in the statistical moments of time-series data as the transition is approached, known as early warning signals (EWSs) [22]–[24], [28]. Early warning signals are measures of the characteristic recovery time of the system and are directly related to the stability of a dynamical regime (Fig. 1). In principle, one only needs to monitor the system behavior in the stable/free flow state to identify how close the system is to an instability (i.e., close to a traffic jam), or how the system stability will respond to the applied traffic management strategies. In this study, we introduce generic early warning indicators of traffic instabilities and assess whether warning signals predict nonlinear transitions from free flow to congested flow on highways when the road conditions deteriorate. The analyses identify candidate statistical measures that are sensitive to changes in the stability and resilience of transportation systems.

Developing methods to predict the risk of traffic congestion in short-term has been considered in previous research. For instance, regressive models have been used for forecasting traffic conditions [15], [29]. Pattern recognition [30]–[32] and neural networks [33]–[35] have also been applied to short-term traffic forecasting. While these methods perform well for certain case-specific conditions, their predictions are less satisfactory for other cases where external changes occur or where the number of historical data is limited [36].

The main advantage of the proposed data-driven early warning signals in this study is that they are not case-specific. Hence, the proposed signals may serve as generalizable indicators, requiring relatively little data from a system to predict its future state. In recent years, there has also been an increased interest in developing metrics to evaluate and monitor transportation systems resilience [37]–[39]. The proposed early warning signals are developed based on universal underlying dynamics of traffic congestions, so they have an improved generality for practical applications. These characteristics suggest that early warning signals may be employed to keep track of changes in the stability and resilience of the traffic flow dynamics, which is of practical interest for performing traffic management. Results of this study advance the monitoring capability, prediction and control of transportation systems where information regarding the status of system stability and resilience is a significant advantage.

II. Methods

A. Traffic Flow Modeling

To simulate data, macroscopic modeling of traffic flows is employed. The resulting equations are solved numerically to reproduce stable and unstable traffic flow behaviors observed in real traffic. Particularly, simulations are performed based on an anisotropic macro speed gradient continuum model developed through the micro-macro linkages of vehicular traffic dynamics [18]. This model comprises two partial differential equations, namely the vehicle density equation $\rho(x,t)$ represented by a continuity equation, and the equation of motion derived from car-following theory, as follows [18]

$$\frac{\delta\rho}{\delta t}+\frac{\delta q}{\delta x}=g\left(x,t\right), \tag{1a}$$

$$\frac{\delta v}{\delta t}+v\frac{\delta v}{\delta x}=\frac{V_e(\rho)-v}{T}+c_0\frac{\delta v}{\delta x}+\xi(x,t), \tag{1b}$$

where $q=\rho v$ is the flow rate of the traffic stream, v is the space mean speed, $\xi(x,t)$ is additive spatiotemporal Gaussian noise with zero mean, white in space and time reflecting the spatiotemporal stochasticity in driver accelerations, and $g(x,t)$ is the generation rate. The generation rate is zero when there is no ramp on the road, and is nonzero otherwise at the location of an in/out ramp $g\left(x,t\right)=q_{in}\delta\left(x-x_{in}\right)-q_{out}\delta(x-x_{out}))$. In this equation, T is the relaxation time, c_0 represents the propagation speed of the disturbances, and V_e is a function representing the relationship between the mean speed and the traffic density under equilibrium conditions, which is selected to be $V_e=v_{max}\left(1/(1+\exp(\frac{k/k_m-0.25}{0.06}))-3.72\times10^{-6}\right)$ [9]. Spatiotemporal stochasticity $\xi(x,t)$ reflects the fact that drivers have error in approximating the road density and adjusting their speed based on the optimal function $V_e(\rho)$. Numerical simulations were carried out using the finite difference method for the following choice of parameters [18]: $v_{max}=30m/s$, $T=10s$, $k_m=0.2veh/m$, $c_0=c_m=11m/s$. To carry out the numerical simulation, we use an upwind integration scheme developed for speed gradient models to discretize the equations, as in the refs. [18], [40], using $\Delta t=1s$ and $\Delta x=100m$ as discretization parameters. The traffic dynamics is then sampled every 20 seconds to generate the results.

The stability of homogeneous traffic flow can be evaluated by analyzing the effect of a change in a selected system parameter, namely a control parameter, on dynamics. The control parameter is a parameter to which the stability of the system is sensitive, and a change in that parameter may move the system towards/away from an instability (Fig. 1). The car density on the road and average velocity of cars moving on the road are examples of control parameters in traffic flow dynamics that affect the flow stability. Assuming the mean density is the control parameter, theoretical stability analysis of the equations of motion shows the homogeneous traffic flow loses its linear stability when the average density of cars on the road crosses two critical densities, namely ρ_{c_1} and ρ_{c_2}, and the homogeneous traffic flow is linearly unstable when $\rho_{c_1}<\rho<\rho_{c_2}$ (see *Appendix*). When the traffic flow is linearly stable, small disturbances to the flow dynamics will disappear, and traffic will return to its homogeneous state. In contrast, in a linearly unstable traffic, disturbances will grow in size resulting in rapid variations in speed and traffic density in an oscillating pattern referred to as a stop-and-go traffic jam. In what follows and for the sake of brevity, the stability (instability) of homogeneous traffic flow state is replaced with traffic flow stability (instability).

The choice of numerical scheme and discretization parameters to solve Eq. (1) can influence the critical densities from the quantitative perspective, but the qualitative dynamics of the flow is preserved [18], [40]. The proposed early warning signals predict the critical points reported from numerical simulations, though, which are $\rho_{c_1}=0.041veh/m$ and $\rho_{c_2}=0.077veh/m$ based on our numerical results; values accurately the same as in previous findings [18], [40].

B. Early Warning Signals

Early warning indicators are statistical indicators obtained from online measurements of the system that reveal proximity to a tipping point. These indicators are developed based on the dynamical system theory and the theory of critical slowing down [22], [24], [26], and are applicable to systems with small fluctuations around their equilibrium state resulting from stochastic perturbations. As a system approaches to a tipping point (e.g., a traffic jam in this study), the system dynamics becomes progressively slow. As a result of this slowing down phenomenon, the system becomes more correlated with its past leading to an increase in autocorrelation estimated from the time series of appropriate system observables (see *Appendix*). Furthermore, perturbations can accumulate, which leads to an increase in the size of the fluctuations and as a result, an increase in variance or other higher-order statistical indicators [22], [41] (see *Appendix*). An increase in variance and lag-1 autocorrelation of fluctuations of the system has been observed in numerous theoretical and experimental complex systems prone to critical transitions [24], [25], [42], [43]. Other indicators have also been explored as early warning signals of impending transitions, including an increased spectral density ratio which is also employed in our analyses [44].

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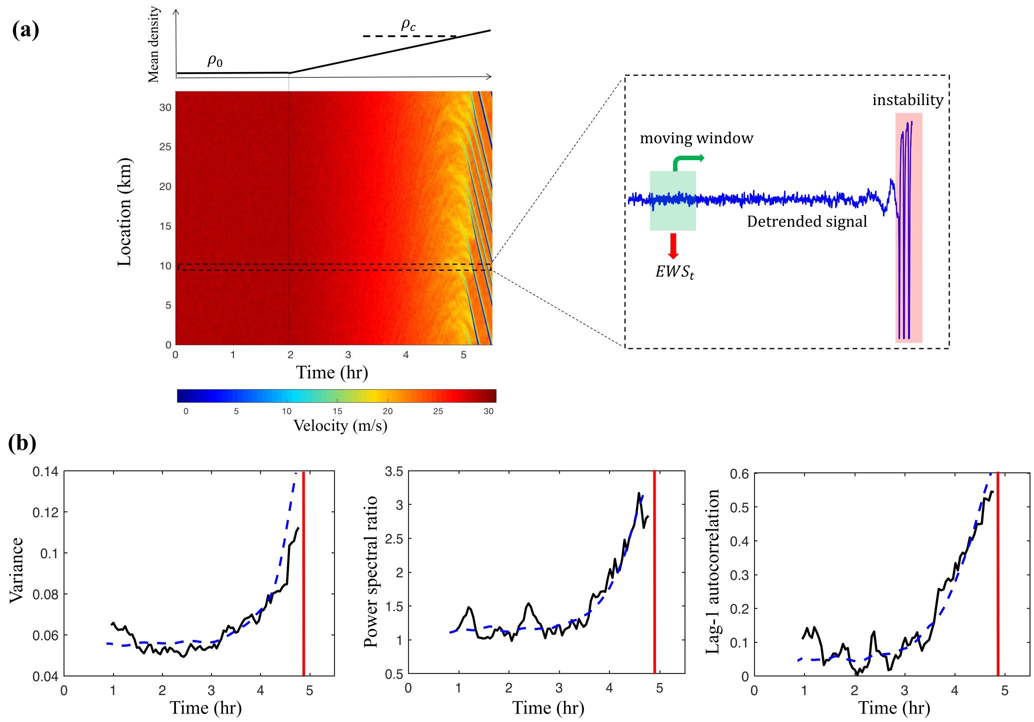

Fig. 2. Early warning signals of traffic instability in a highway with periodic boundary conditions. (a) Spatiotemporal pattern of the traffic flow velocity under a change in the mean density. The mean car density on the road is stationary for the first simulated two hours and increases linearly afterward to reach to its critical value. Patterns of upstream stop and go traffic jam are observed when the dynamics passes its stability threshold. (b) Early warning signals of loss of the traffic stability and resilience obtained from stochastic fluctuations in the measured mean velocity of cars at selected road segments. In each graph, the solid black line is the EWS obtained from a single $500m$ segment of the road, the dashed blue line is the average of the EWSs obtained from all road segments, and the vertical red line shows the time at which the traffic instability is observed.

C. Approximating Early Warning Signals From Traffic Flow Measurements

To search for indications of critical slowing down prior to the traffic instability, we assess the recorded stochastic fluctuations of the measured traffic variables and calculate early warning statistics from the recorded time series. As demonstrated in Fig. 1(b), the selected observables of the traffic flow dynamics (e.g. mean density/velocity of cars) are measured online at each time instant. Trends in early warning indicators are estimated using a moving window. Data at each window are detrended, and the variance, lag-1 autocorrelation and spectral density ratio of the recorded measurements are extracted. The spectral density ratio at each window is defined as the spectral power in the lowest 20% to the highest 20% portion of spectral frequencies in the window. The moving window will then shift in time to include the new measurements and the early warning signals are instantly computed in the updated window. As a result, the trend of early warning signals is observed online as more measurements are gradually added to the time series (Fig. 1(b)). Strong positive trends in early warning signals are expected from the theoretical studies once the traffic approaches an instability (see *Appendix*).

In addition to monitoring single indicators, to determine whether a composite signal would provide an earlier and more accurate warning of the approaching instability, we calculate composite indicators based on the sum of the standardized deviations of each statistic from its long run average [24]. The time series for each statistic (w_i) is normalized at every time point by subtracting the long-run average of that indicator dividing it by the long-run standard deviation, i.e.,

$$\hat{w}_{i,t} = \frac{w_{i,t} - \frac{1}{n}\sum_{\tau=1}^{t} \mathrm{w}_{i,\tau}}{sd(w_{i,\tau})}, \tag{2}$$

where n is the number of samples from times 1 to t, and $sd(w_{i,\tau})$ is the standard deviation over the same period. The composite indicator is then defined as $W_t = \sum_{i=1}^{p} \hat{w}_{i,t}$, and p is the number of incorporated indicators in the composite signal. The composite early warning index is considered to produce an early warning signal when the value of the composite index exceeds its running mean by two standard deviations [24]. In this study, we show whether and how combining incorporating single indicators to a composite indicator improves the performance of warning signals in predicting an impending transition. Note that the composite indicator and its corresponding 2σ threshold are not derived from the theory of slowing down. This method quantifies how significant an indicator or a combination of indicators change relative to the reference condition when no sign of slowing down exists in the time series.

III. Results

Theoretical findings and tests on simulated and empirical case studies reveal that the slowing down phenomenon in the traffic flow dynamics is detectable prior to its linear stability boundary. We assessed the performance of three time-varying early warning signals, namely variance, lag-1 autocorrelation,

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and spectral density ratio extracted from measured stochastic fluctuations of the observed flow dynamics on highways. Our main finding is that each of the introduced indicators shows a substantial increase as the flow loses its stability and resilience and the system approaches its critical condition, i.e. a traffic jam. In addition to warning impending instabilities, early warning metrics are shown to be sensitive to changes in the stability and resilience of the traffic dynamics making them a potential tool to evaluate the effectiveness of adopted management strategies.

A. Results of a Study Using Simulated Data

Early warning signs of traffic instabilities are introduced and evaluated in two simulated traffic scenarios: (i) traffic jam on a closed road loop without a bottleneck, and (ii) traffic jam on an open-road with ramps. The analyses are based on macroscopic traffic flow dynamics, neglecting individual vehicle dynamics and focusing on the average properties of flow dynamics on the considered road segment. For the traffic flow to approach an instability, a deteriorating condition needs to be modeled so that the system is progressively pushed toward a transition (traffic jam) over time (Fig. 1). Such a deteriorating condition is characterized by a control parameter, which can be any single factor affecting the traffic dynamics, e.g. average car density on the road, the time of day, the weather conditions, driving strategies, or a combination of them that changes over time and pushes the system toward an instability. In the following examples, we selected a variety of control parameters including the average car density on the road, maximum speed limit and the traffic inflow to the road. However, the methods discussed are more general, and may be applied to cases with other known/unknown underlying causes of instability.

The closed loop road is modeled as a road segment with periodic boundary conditions. Assuming the mean density is the control parameter and for the range of parameters considered in the simulation, a stop-and-go traffic appears when the mean car density on the road progressively increases from values $\rho < \rho_{c_1}$ toward the critical value of $\rho_{c_1} = 0.041 veh/m$. The flow simulation starts at an initial mean road density of $\rho_0 = 0.01$ in the free flow equilibrium condition, a value sufficiently far from the critical value, with spatiotemporal stochasticity added to the driver accelerations. After an initial phase of stationary simulations at ρ_0 reflecting steady condition on the road, the mean density is gradually increased toward ρ_{c_1}. To extract early warning signals, we monitor selected road segments of the length 500 m and monitor the stochastic fluctuations in the dynamical features of the flow over time. The potential traffic dynamics variables to be monitored are the mean velocity and the mean density of cars in the selected road segment over time. In what follows, for the sake of brevity, we presented results only for measured velocities. Similar approach can be followed by measuring the density of vehicles on road. For any simulated trajectory, we calculated and assessed the performance of early warning signals extracted from measured stochastic fluctuations of mean vehicle speeds on the selected road segments (Fig. 2). Results show that the introduced indicators exhibit a substantial increase as the mean density of cars on the road approaches its critical value and the system approaches a traffic jam.

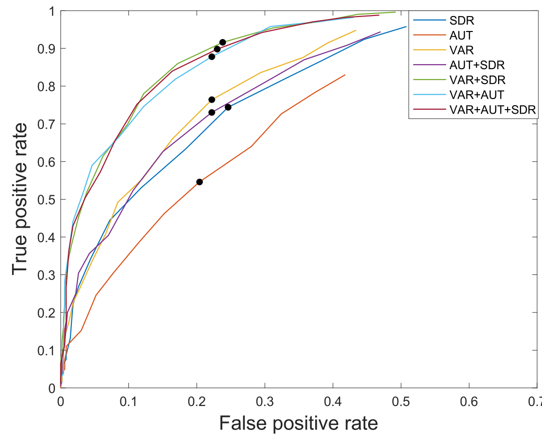

Fig. 3. Receiver Operator Characteristics (ROC) for the seven early warning signal metrics. The curves were constructed by 1000 independent simulations of the traffic flow equations, half of which exhibiting instability in their dynamics. Curves are constructed by varying the number of the consecutive signals above the 2σ threshold as the sign of an imminent transition. Solid circles on the curve show the location corresponding to five consecutive signals above the 2σ threshold.

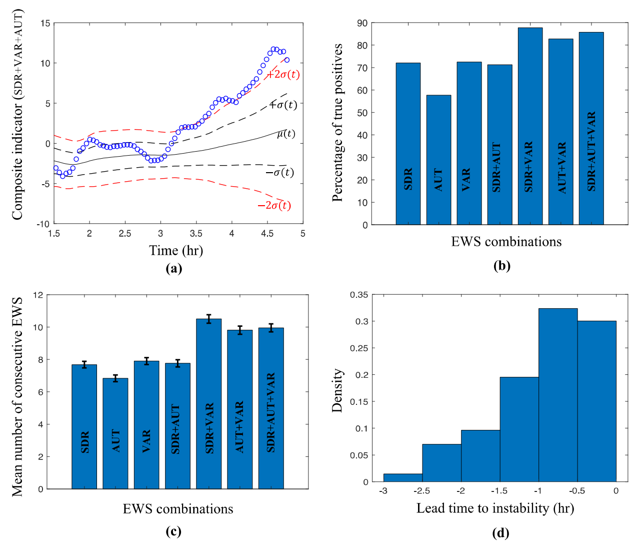

Fig. 4. (a) Example of the constructed composite signal combining standardized variance (Var), lag-1 autocorrelation (AUT) and spectral density ratio (SDR) plotted against time. The blue circles show the composite signal, and the running average (μ) and standard deviation boundaries (σ and 2σ) are also marked in the figure. A passage of the 2σ threshold by the composite index represents a slowing down in the dynamics. (b) Percentage of the true positives, i.e. the composite index passes the 2σ threshold for five consecutive time steps, for each combination of the EWSs obtained from 400 independent simulations of the traffic flow dynamics exhibiting instability. (c) Mean number of maximum consecutive passages of the 2σ threshold for each combination of EWSs. (d) Distribution of the remaining time to a traffic jam when the composite indicator $W = Var + AUT + SDR$ predicts an imminent instability based on the 2σ criteria.

Although one observes a progressive increase in the values of the warning signals as the system loses its stability and resilience, we investigated whether a composite signal would provide a more accurate and stronger warning of the approaching transition. To this aim, in line with previous work by Drake and Griffen in ecological systems [24], we normalized each indicator independently and constructed a composite

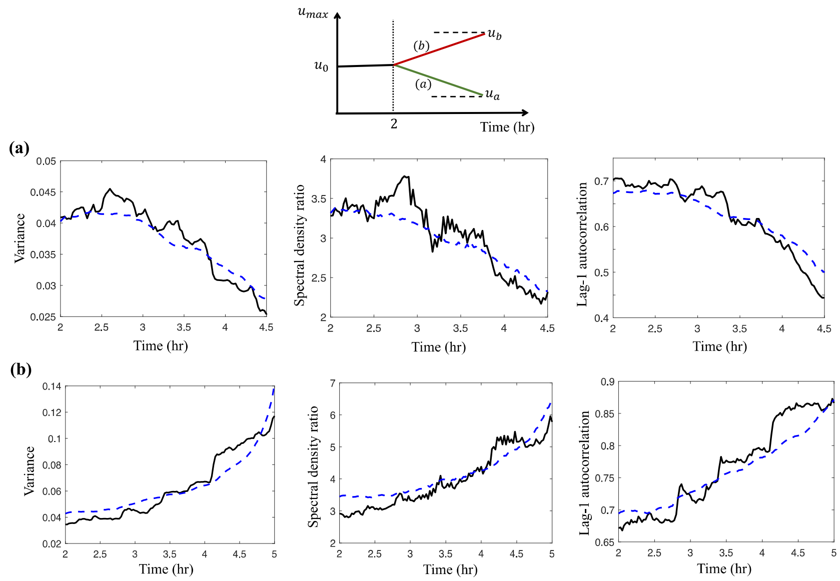

Fig. 5. Measurable changes in the trend of the early warning signals at a fixed mean density and a progressive (a) decrease and (b) increase in the maximum allowed driving speeds. The solid black lines represent the EWSs obtained from a single $500m$ segment of the road, and the dashed blue lines show the average of the EWSs obtained from all road segments.

indicator combining multiple indicators into a single metric based on the sum of the standardized individual indicators. The resulting composite signal is considered to be unequivocal when its value exceeded its running average by two standard deviations (2σ), reflecting a significant slowing down and risk of transition in the dynamics. We simulated 400 independent trajectories of the traffic flow dynamics starting from the mean density of ρ_0 progressively increased toward the critical value ρ_{c_1}, and constructed the composite index for all possible combinations of indicators to compare the performance of single indicators and composite indicators in anticipating the risk of an impending instability (i.e., a traffic jam). Thus, in total we tested seven different metrics composed of every unique combination of one to three indicators. For each case, we computed the fraction of the cases where the signal passes the 2σ threshold and stays above the threshold for five consecutive time steps, selected based on the constructed receiver operating characteristic (ROC) curve for the parameters selected in this simulation (Fig. 3). Note that this number is obtained for simulation parameters and might need to be adjusted for empirical applications. The results of this analysis are shown in Fig. 4. Interestingly, we found that composite warning signals composed of some or all metrics outperform the accuracy of any individual warning signal. An elaborately selected composite index would provide more accurate and significant warning signal of traffic instability compared to an individual indicator (Fig. 4). Based on the results presented in Figs. 3 and 4, the accuracy of predicting an upcoming traffic congestion is close to 90% if a composite warning signal is selected considering the defined 2σ criteria.

In addition to evaluating the risk of a transition, early warning signals are a potential tool to evaluate changes in the dynamics of the system caused by any known/unknown source. Particularly, the signals can be employed also as a tool to evaluate the effects of adopted traffic management strategies on the traffic stability. We demonstrate this by investigating the effect

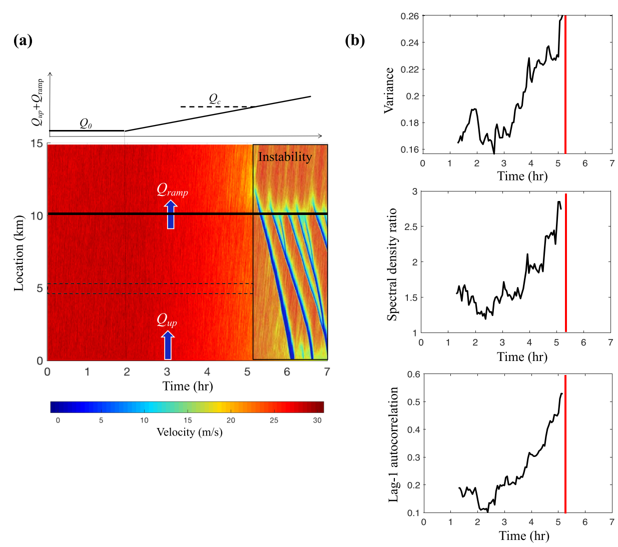

Fig. 6. Early warning signals of traffic instability in a highway with an on-ramp. (a) Spatiotemporal pattern of the traffic flow velocity under the change of the inflow to the road. The traffic inflow is stationary for the first simulated two hours and increases linearly afterward to reach its critical value. (b) Early warning signals of loss of the traffic stability and resilience obtained from stochastic fluctuations in the measured mean velocity of cars at selected road segments (schematically shown with a horizontal box in the figure). The vertical red line shows the time at which the traffic instability is observed.

of changes in the maximum speed limit on the stability of the traffic flow dynamics. To this aim, we set the initial mean car density of the road ρ_0 and the maximum driver velocity v_{max} so that the traffic is at a close distance to the tipping point. Keeping the mean road density fixed, the maximum allowed speed for vehicles was gradually increased. Changes in the maximum driving speed can resemble a policy defined for the system by traffic managers, or a consequence of any changes in the system affecting driving strategies (e.g., time of the day, weather). For such a simulation, results reveal a clear and marked change (decrease) in the system stability after

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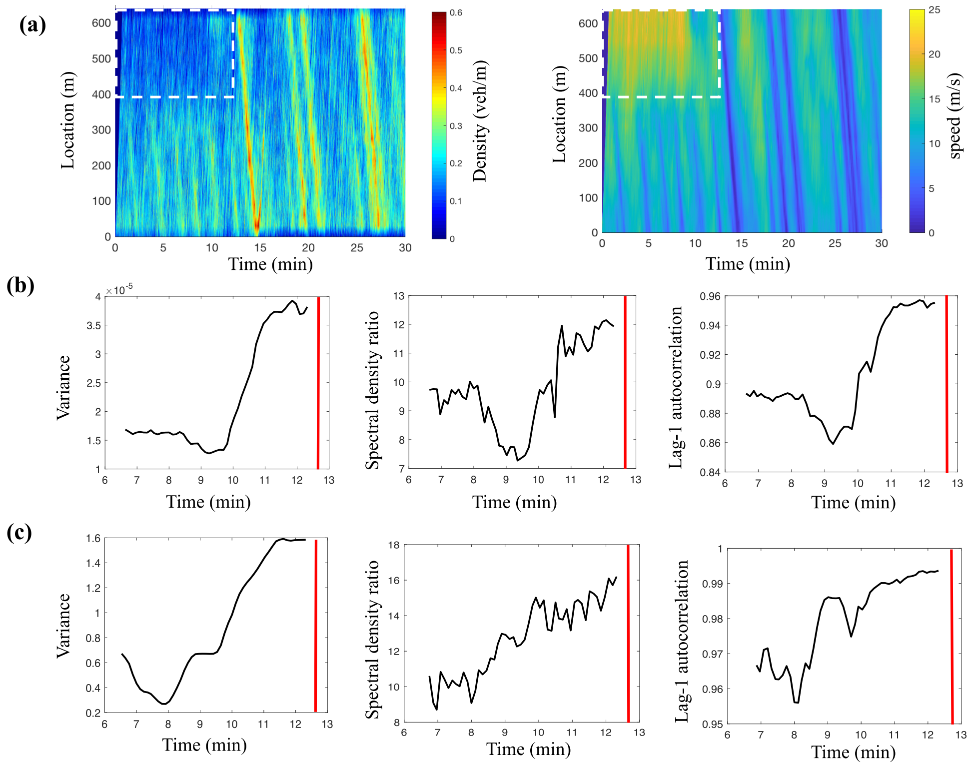

Fig. 7. Early warning signals of traffic congestions for US-101 highway. (a) Spatio-temporal evolution of density and velocity of vehicles on the road for the first 30 minutes of the recordings. The measurements used to extract early warning signals is depicted by a dashed box on the plots. (b) Early warning signals obtained from density measurements. (c) Early warning signals obtained from velocity measurements. Vertical red line shows the time at which the traffic instability is observed. (d) Composite early warning index comprising all three indicators extracted from velocity measurements (left) and density measurements (right) in the empirical US-101 highway case study.

the change in the maximum allowed speed starts (Fig. 5). Hence, monitoring early warning signals shows how and if the stability of the system has changed significantly as a result of a change in the system parameters. A similar study was performed by gradually decreasing the maximum possible driving speed, meaning that the drivers progressively tend to drive slower compared to their initial assigned value (Fig. 5). The measured warning signals in this case exhibit a decrease, representing an increased stability and resilience of the traffic as a result of this change in the driving strategy.

As a second study, we next performed an analysis of the macroscopic model of an open freeway with an on-ramp. The stability of this system is a function of both the variation in the inflow at the upstream freeway boundary (Q_{up}) and at the ramp (Q_{ramp}) [45]. The upstream traffic can lose its resilience and approach an instability by increasing either or both of these parameters. Here, we assume both the upstream and ramp inflows increase progressively over time starting from an initial value. The simulated freeway has a length of 15 km and a ramp located at $x = 10$ km. In line with the previous example, we monitor the mean vehicle speeds recorded at selected road segments of length 500 m located upstream of the ramp.

The stability of the traffic is evaluated using the introduced early warning statistics (i.e. variance, lag-1 autocorrelation, spectral density ratio) over time as the inflow to the road is gradually increased. Figure 6 shows the sensitivity of the introduced early warning indicators to the changes in the stability of traffic dynamics on this highway. All the introduced indicators respond to a change in the traffic inflow into the highway, which implies a reduction in the system stability and resilience to perturbations. A significant increase in the warning signals is observed when the system is close to the instability compared to the baseline values corresponding to the stationary phase of the dynamics.

B. Results of a Study Using Empirical Data

To test the performance of the prediction/detection algorithm, we carried out a study using empirical time series collected by the Next Generation Simulation Program from the United States Federal Highway Administration [46]. Here, we study the data recorded on the southbound of freeway US-101 in Los Angeles in 2005 since it exhibits a traffic instability after a period of initial free flow regime. A total of 45 minutes of data in morning rush hours is available which includes both free flow and traffic jam propagation states. To capture the steady and patterns and to avoid inactive cells in spatiotemporal profiles, the first and last $100s$ in the temporal dimension and $100ft$ in the spatial dimension were removed from the analysis. The data from the auxiliary lane and ramps were eliminated from the analysis because the normal traffic in these lanes differ from that found in the other lanes.

Figure 7 shows the spatio-temporal profile of traffic flow constructed from the recorded data. The plot is obtained by partitioning the road in space and time neglecting the effect of lane changing and the microscopic dynamics of vehicles. Analyzing the dataset shows that although a stop-and-go traffic

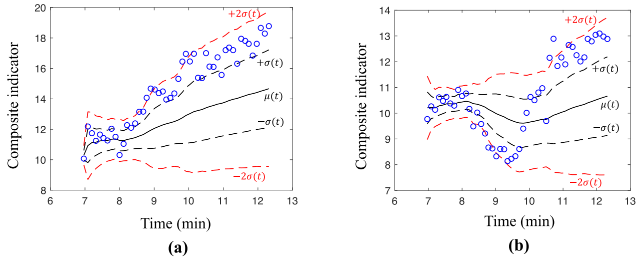

Fig. 8. Composite early warning index comprising all three indicators extracted from velocity measurements (left) and density measurements (right) in the empirical US-101 highway case study.

exists in the first 400 m of the road from the beginning of the measurements, the traffic is stable in the last 200 m of the road for the first 15 minutes of the recordings. As a result, we focus on the last 200 m of the road to search for signs of an upcoming instability using early warning signals. To identify EWSs of traffic instability, both the spatial-mean velocity of vehicles and the mean density of the vehicles on the monitored road segment over time are measured. We focus on the initial part of the data up to the emergence of the first instability on the road (i.e., $t \approx 12.8\ min$) to evaluate the performance of the introduced warning signals. The extracted warning indicators are depicted in Fig. 7, showing a measurable rise in the amplitudes of the warning indicators before the onset of the instability on the road. Among the measured indicators, the variance provides a more robust warning signal. Using the approximated warning signals, we constructed the composite early warning index comprising all three indicators to assess the performance of composite indicator in this empirical study. Results show that the composite indicator passes the 2σ threshold, and warns of an impending transition measuring either the velocity or the density fluctuations (Fig. 8). It is observed that the composite indicator fluctuates around the 2σ boundary for several minutes prior to the onset of instability and maximum three consecutive signals are detected above the threshold.

IV. Discussion and Conclusion

We approach the problem of data-driven traffic jam prediction and evaluation from complex dynamical systems and critical transitions point of view. We suggest that generic early warning indicators developed based on dynamical systems theory offer a potential solution to the formidable challenge of traffic stability and resilience evaluation. Due to the computational efficiency of early warning signals, they provide a feasible tool for online stability analysis of transportation systems. We demonstrated that early warning signals can successfully alarm approaching a traffic instability in both numerical and empirical case studies. Although each of the indicators independently showed evidence of approaching the transition, we found that a composite early warning index comprising all or some of the single indicators could be more efficient in detecting slowing down and potential transitions, leading to an increased sensitivity and reliability of predictions and a reduced chance of a false alarm. In addition to warning impending instabilities, early warning metrics were shown to be capable of detecting changes in the underlying system parameters and the degree they affect the stability of the flow, which is a formidable challenge in the absence of a detailed and calibrated model of system components and interactions. The proposed approach is model-free and requires relatively little data or understanding of the underlying structure of a system. Of course, warning signals are beneficial for systems that might be at risk of instabilities and critical transitions. The potential for instabilities could come from observations of the history of the system dynamics or from simplified and low-fidelity models. However, once the system is identified to be at risk of critical transitions, the introduced warning signals can be applied to evaluate the system stability and resilience regardless of availability of detailed information about underlying system parameters and equations. This feature addresses one of the significant obstacles in complex transportation systems, which is the reliance on highly parameterized models that are prone to mis-specification and may give little or no indication of qualitative changes in system dynamics [10], [11].

The empirical case study supports the potential applicability of introduced warning indicators to assess changes in the stability and resilience of real-world transportation systems. It should be noted that the time series used for the empirical case study (and more likely other real-world data) are shorter than the scenarios presented in the simulations and contain more uncertainties from unknown sources. As a result, the extracted warning signals are more susceptible to data processing steps [47], [48]. Therefore, an initial study on the optimal data processing steps might be required in real-world scenarios.

Despite the advantages of the early warning signals and critical slowing down based indicators, there are limitations associated with such methods that need to be taken into consideration while interpreting the results. First, early warning signals are applicable only to systems that exhibit certain types of instabilities in their dynamics. Mathematically, for continuous time systems these instabilities are observed when the real part of dominant eigenvalue of the dynamics gradually approaches zero. The underlying mechanism causing an instability, particularly traffic jam in this study, is required to satisfy this assumption for early warning signals to work. Second, some systems do not exhibit slowing down in their dynamics until they are in a narrow neighborhood of the instability. As a result, although observing a sign of slowing down means the system is losing stability, failure to detect slowing down does not necessarily imply the stationarity of the system dynamics. Third, strong environmental noise may erase signatures of critical slowing down. Particularly when the system is bistable, strong noise or perturbations can force a system to another state far before it reaches the tipping point and experiences a slowing down in its dynamics [49], [50]. In addition, early warning indicators predict approaching an instability, but do not provide a quantitative measure of how close (in time or control parameter) the system is to the instability. Such an approximation depends on many system specific factors, e.g. the rate of change in the control parameter, system dynamics parameters, which are not considered in this study.

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Although the slowing down phenomenon and the resulting increase in early warning indicators sound the alarm regarding the risk of loss in stability and resilience, one should note that early warning indicators are a direct measure of changes in the stability but not necessarily the resilience. By definition, stability represents the ability of a system to return to its equilibrium state after a temporary disturbance [26], [51]. The more rapidly the system returns to its equilibrium, the more stable it is. In contrast, resilience is a measure of the magnitude of disturbance that can be absorbed before the system switches to another equilibrium state [43], [51]. Early warning indicators are developed based on the slowing down phenomenon, i.e. decreased rate of recovery from perturbations as it approaches an instability. As a result, they are directly related to the system stability. Although an increase in the trend of the indicators implicitly raises the alarm for a potential loss of the resilience, it does not identify how and whether the resilience is decreased. For instance, the system might approach a supercritical type of bifurcation at which the system approaches an instability, but it can recover from perturbations of any amplitude and the resilience is retained before the tipping point. In subcritical bifurcations (Fig. 1(a)), however, loss of stability and resilience are related when the system is in the bistable region of the dynamics. In this case, resilience becomes an important factor since the system might experience instability even before the tipping point. Subcriticality and bistability are widely observed in the literature of transportation systems, particularly for highway traffic where large perturbations in the dynamics result in the emergence of a traffic jam whilst the traffic has not reached its linear stability boundary [45], [52], [53]. Hence, once a rise in the early warning indicators of a transportation system is observed, one should expect the loss of both the stability and resilience to remain on the safe side, implying that the system has become more vulnerable to jumping to a congestion state earlier than expected if a large disturbance occurs on the road.

The concepts and methods introduced in this study were applied to the traffic congestion scenario on highways. The introduced metrics, however, are generalizable, as they make limited assumptions about the underlying complex dynamics, infrastructure and environmental parameters affecting the system. The approaches and ideas can be used as a basis for further development to address challenges related to the complexity of traffic dynamics.

Appendix

A. Linear Stability Analysis of the Equations

Consider the traffic flow model governed by the following equations [18]

$$\frac{\delta\rho}{\delta t} + \frac{\delta q}{\delta x} = g(x,t), \tag{3a}$$

$$\frac{\delta v}{\delta t} + v\frac{\delta v}{\delta x} = \frac{V_e(\rho) - v}{T} + c_0\frac{\delta v}{\delta x}. \tag{3b}$$

Assume ρ_s and $v_s = V_e(\rho_s)$ to be the uniform flow solution of these equations for $g(x,t) = 0$. The equilibrium state of a dynamical system is a state that a system returns to it after a small temporary disturbance. Assuming periodic boundary conditions, we define the perturbed solutions of the equations around the uniform flow equilibrium solution as $\rho(x,t) = \rho_s + \tilde{\rho}_\gamma \exp(i\gamma x + \sigma_\gamma t)$ and $v(x,t) = v_s + \tilde{v}_\gamma \exp(i\gamma x + \sigma_\gamma t)$. Substituting these solutions into Eqs. (3a) and (3b), taking a Taylor series expansion of the nonlinear terms, and neglecting higher order terms one obtains

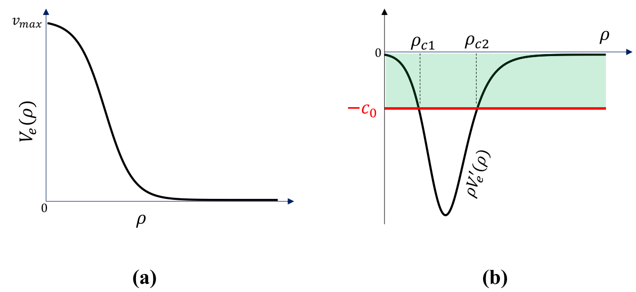

Fig. 9. Schematic of (a) the fundamental velocity function (V_e) and (b) the resulting stability boundaries obtained in Eq. (5). The stable region is marked with green color.

$$\left(\sigma_\gamma + i\gamma v_s\right)^2 + \left(\frac{1}{T} - i\gamma c_0\right)\left(\sigma_\gamma + i\gamma v_s\right) + \frac{1}{T}\left(i\gamma\rho_s\right)V_e'(\rho_s) = 0. \tag{4}$$

where $V_e'(\rho_s) = \frac{dV_e}{d\rho}(\rho_s)$. Substituting $\sigma_\gamma = \alpha + i\beta$ into Eq. (4), separating real and imaginary parts of the resulting equations and performing some straightforward algebra, one reaches the following stability condition corresponding to $\alpha = 0$:

$$-c_0 \leq \rho_s V_e'(\rho_s) \leq 0. \tag{5}$$

Therefore, the uniform traffic becomes unstable and the system exhibits a traffic congestion when either $\rho_s V_e'(\rho_s) + c_0 = 0$ or $V_e'(\rho_s) = 0$. For the common velocity function considered in the literature and in this study [9], [54] (see *Methods*), V_e is a monotonically decreasing function of ρ, and the condition $V_e'(\rho_s) = 0$ corresponds to the extreme limits of the dynamics ($\rho_s = 0$ or $\rho_s \to \infty$). Thus, our focus is on the instability caused by the $\rho_s V_e'(\rho_s) + c_0 = 0$ criteria. This stability criteria corresponds to two critical values of density, namely ρ_{c1} and ρ_{c2}, where $\rho_{c1} < \rho_{c2}$ (Fig. 9). The analyses performed in the main text focused on the critical density of ρ_{c1} which is a more common type of instability on roads, but one can obtain similar results by evaluating the proposed methods for the critical transition at ρ_{c2}.

B. Variance Analysis

We derived an analytical expression for the variance of the measured mean velocity and density of the cars on road to analyze their trend as the traffic approaches an instability. Starting from the following equations

$$\frac{\delta\rho}{\delta t} + \frac{\delta q}{\delta x} = g(x,t),$$
$$\frac{\delta v}{\delta t} + v\frac{\delta v}{\delta x} = \frac{V_e(\rho) - v}{T} + c_0\frac{\delta v}{\delta x} + \xi(x,t), \tag{6}$$

where $\xi(x,t)$ is additive spatiotemporal Gaussian noise with zero mean, white in space and time, and correlation given by

$\langle\xi(x,t)\xi\left(x',t'\right)\rangle = r^2\delta\left(x-x'\right)\delta(t-t')$ where r is the noise intensity. Assuming $g\left(x,t\right)=0$, and ρ_s and $v_s = V_e(\rho_s)$ to be the steady state solutions of the above equations, we define the perturbed solutions of the equations around the uniform flow equilibrium solution as $\rho\left(x,t\right)=\rho_{\mathrm{s}}+k(x,t)$ and $v\left(x,t\right)= v_{\mathrm{s}}+u(x,t)$. Substituting the perturbed solutions into Eq. (6), taking Taylor series expansions of the perturbed equations at ρ_{s} and v_{s}, and neglecting higher order terms, we obtain the following linearized equations:

$$\frac{\delta k}{\delta t}+v_s\frac{\delta k}{\delta x}+\rho_s\frac{\delta u}{\delta x}=0,$$
$$\frac{\delta u}{\delta t}+v_s\frac{\delta u}{\delta x}=\frac{V_e'(\rho_s)k-u}{T}+c_0\frac{\delta u}{\delta x}+\xi(x,t), \qquad (7)$$

where $V_e'\left(\rho_s\right)=\frac{dV_e}{d\rho}(\rho_s)$. Assuming periodic boundary conditions, we can consider the solution to Eq. (7) in the following form:

$$u\left(x,t\right)=\sum\nolimits_{\gamma}e^{i\gamma x}u_{\gamma}\left(t\right),$$
$$k\left(x,t\right)=\sum\nolimits_{\gamma}e^{i\gamma x}k_{\gamma}\left(t\right). \qquad (8)$$

Taking the Fourier transform of the linearized Eq. (7) and separating real and imaginary functions, four independent coupled ordinary differential equations are obtained as follows:

$$\dot{K}_{\gamma R}-\gamma v_s K_{\gamma I}-\gamma\rho_s U_I=0,$$
$$\dot{K}_{\gamma I}+\gamma v_s K_{\gamma R}+\gamma\rho_s U_R=0,$$
$$\dot{U}_{\gamma R}-\gamma v_s U_{\gamma I}=\frac{1}{T}\left[V_e'\left(\rho_s\right)K_{\gamma R}-U_{\gamma R}\right]$$
$$-c_0\gamma U_{\gamma I}+\xi_{\gamma}\left(t\right),$$
$$\dot{U}_{\gamma I}+\gamma v_s U_{\gamma R}=\frac{1}{T}\left[V_e'\left(\rho_s\right)K_{\gamma I}-U_{\gamma I}\right]+c_0\gamma U_{\gamma R}, \quad (9)$$

where $u_{\gamma}\left(t\right)=U_{\gamma R}+iU_{\gamma I}$ and $k_{\gamma}\left(t\right)=K_{\gamma R}+iK_{\gamma I}$. $\xi_{\gamma}\left(t\right)$ is the noise term obtained by taking the Fourier transform of $\xi(x,t)$ and is Gaussian, white in time, independent for distinct wavenumbers γ, with amplitude $\beta(\gamma)$ determined by spatial power spectral density. Equation (9) can be written in matrix form as

$$d\mathrm{x}_{4\times1}=A_{4\times4}\mathrm{x}_{4\times1}dt+L_{4\times1}dw(t), \qquad (10)$$

where $\mathrm{x}=\left[K_{\gamma R},K_{\gamma I},U_{\gamma R},U_{\gamma I}\right]^T$, and $w(t)$ denotes the standard Brownian motions, and $L=[0,0,\beta,0]^T$ with β being a constant. The equations for the covariance matrix of the Eq. (10) is

$$\dot{P}=AP+PA^T+LL^T. \qquad (11)$$

The steady state solution of Eq. (11) (i.e., $\dot{P}=0$) provides the covariance matrix of the equations where its diagonal and off-diagonal elements are the variances and covariances of the state vector x, respectively. To solve this equation, we employed a matrix Kronecker product approach [55] to transform the equation into the equivalent system of linear equations and found the elements of covariance matrix P by solving the resulting equations symbolically using Mathematica®[56]. Following this procedure, one can find the

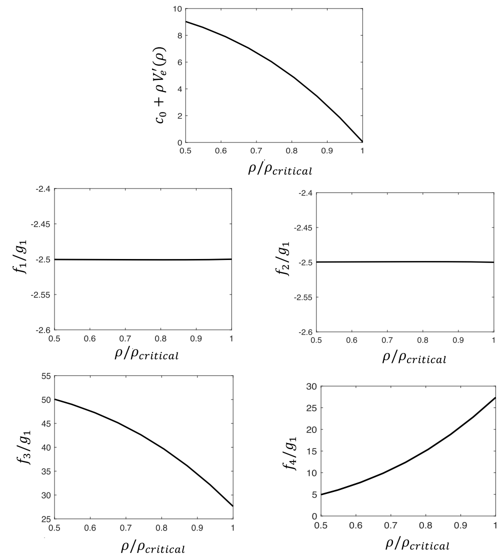

Fig. 10. Trends of the functions appeared in Eq. (12) computed for the model parameters described in *Methods* and varying ρ as the control parameter.

expressions for the variances

$$var\left(K_{\gamma R}\right)=\frac{\rho_s\beta f_1\left(\gamma,c_0,T,\rho_s,v_s,V_e'\left(\rho_s\right)\right)}{V_e'\left(\rho_s\right)\times\left(c_0+\rho_s V_e'\left(\rho_s\right)\right)\times g_1\left(\beta,\gamma,c_0,T,\rho_s,v_s,V_e'\left(\rho_s\right)\right)},$$
$$var\left(K_{\gamma I}\right)=\frac{\rho_s\beta f_2\left(\gamma,c_0,T,\rho_s,v_s,V_e'\left(\rho_s\right)\right)}{V_e'\left(\rho_s\right)\times\left(c_0+\rho_s V_e'\left(\rho_s\right)\right)\times g_1\left(\beta,\gamma,c_0,T,\rho_s,v_s,V_e'\left(\rho_s\right)\right)},$$
$$var\left(U_{\gamma R}\right)=\frac{\beta f_3\left(\gamma,c_0,T,\rho_s,v_s,V_e'\left(\rho_s\right)\right)}{\left(c_0+\rho_s V_e'\left(\rho_s\right)\right)\times g_1\left(\beta,\gamma,c_0,T,\rho_s,v_s,V_e'\left(\rho_s\right)\right)},$$
$$var\left(U_{\gamma R}\right)=\frac{\beta f_4\left(\gamma,c_0,T,\rho_s,v_s,V_e'\left(\rho_s\right)\right)}{\left(c_0+\rho_s V_e'\left(\rho_s\right)\right)\times g_1\left(\beta,\gamma,c_0,T,\rho_s,v_s,V_e'\left(\rho_s\right)\right)}. \qquad (12)$$

The expressions derived in Eq. (12) provide an insight into the expected trend of the early warning signals obtained by monitoring the mean velocity and density of cars on the road. Recall from the stability analysis that the traffic exhibits an instability when $c_0+\rho_s V_e'\left(\rho_s\right)=0$, which results into the slowing down of the dynamics as the system loses its stability. This term exists in the denominator of the variances extracted from both velocity and density measurements, pointing to the fact that the variances grow indefinitely when the system is extremely close to the instability. Trends of the functions $f_i, i=1,2,3,4$, and g_1 in Eq. (12) are demonstrated in Fig. 10 highlighting that the term $c_0+\rho_s V_e'\left(\rho_s\right)$ dominates the equations for densities sufficiently close to the $\rho_{critical}$.

C. Early Warning Signals

Consider a dynamical system

$$\dot{z}=f\left(\mu,z\right)+\sigma\eta, \qquad (13)$$

where f is a nonlinear function, μ is the control (varying) parameter, z is the state variable, and $\eta(t)$ represents stochastic white noise with variance σ^2. Assuming the system is at its equilibrium state (i.e., z^*), one can expand Eq. (13) to the first order to obtain

$$\dot{\tilde{z}} = \lambda(\mu)\tilde{z} + \sigma\eta, \tag{14}$$

where $\tilde{z} = z - z^*$ and $\left.\frac{df(\mu,z)}{dz}\right|_{z^*} = \lambda(\mu)$. The system is stable when $\lambda(\mu) < 0$ and becomes unstable when $\lambda(\mu) > 0$. The parameter $\mu = \mu_c$ at which $\lambda(\mu_c) = 0$ is the critical parameter value, also referred to as the tipping point of the system.

The variance for the time series $\tilde{z}(t)$, is $< \tilde{z}^2 >$, where $\langle\cdot\rangle$ denotes statistical mean. From Eq. (14), one can show that

$$< \tilde{z}^2 > = \frac{\sigma^2}{2\lambda(\mu)}. \tag{15}$$

The autocorrelation function of time series $\tilde{z}(t)$ is $c(\Delta t) = < \tilde{z}(t)\tilde{z}(t + \Delta t) >$, where Δt represents time lag. From Eq. (14), a closed form expression of normalized autocorrelation function can be derived as

$$K = \frac{c(\Delta t)}{< \tilde{z}^2 >} = \exp(\lambda(\mu)|\Delta t|). \tag{16}$$

The coefficient $K = \exp(\lambda(\mu)|\Delta t|)$ where $\Delta t = t_{j+1} - t_j$ and j is the sample index represents the autocorrelation at lag-1. Assuming that there is a repeated disturbance of the state variable after each period Δt (i.e., the additive noise), the return to equilibrium is approximately $\exp(\lambda(\mu)|\Delta t|)$.

Assuming that a system is initially at a sable state with $\lambda(\mu) < 0$ and it gradually loses its stability, i.e. $\lambda(\mu) \to 0^-$, Eq. (15) shows that the variance of the time series increases and approaches to infinity when $\lambda(\mu_c) = 0$. In addition, Eq. (16) shows that K approaches 1 exponentially when $\lambda(\mu) \to 0^-$. As a result, monitoring the variance and lag-1 autocorrelation of the time series may be selected as indicators of approaching an instability. Beside these two indicators, other statistical metrics based on slowing down phenomena have been introduced also as potential warning signals. In particular, the spectral density ratio was shown to increase while the system approaches the critical threshold since low frequency directional movement begins to dominate the high frequency noise-induced fluctuations [49]. In this study, we analyze the stability and resilience of traffic flow system using the variance, lag-1 autocorrelation, and spectral density ratio of the measured time series.

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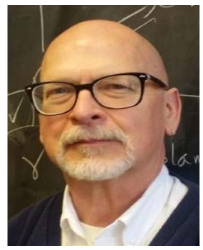

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